Detecting and measuring faint point sources with a CCD

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Stars, Asteroids, and even the (pseudo-)nuclei of comets, are point-sources of light. In recent times, most observers use CCDs to observe these objects, so it might be worthwhile to think about some details of detecting and measuring point sources with a CCD. First, this paper discusses the properties of point-sources, and how they can describe them with a small set of numerical values, using a Point Spread Function (PSF). Then, the sources of noise in CCD imaging systems are identified. By estimating the signal to noise ratio (SNR) of a faint point source for some examples, it is possible to investigate how various parameters (like exposure time, telescope aperture, or pixel size) affect the detection of point sources. Finally, the photometric and astrometric precision expected when measuring faint point sources is estimated.

Introduction

Modern CCD technology has enabled amateur astronomers to succeed in observations that were reserved to professional telescopes under dark skies only a few years ago. For example, a 0.3m telescope in a backyard observatory, equipped with a CCD, can detect stars of 20^{mag}. However, many instrumental and environmental parameters have to be considered when observing faint targets.

Properties of Point Sources

In long exposures, point sources of light will be "smeared" by the effects of the atmosphere, the telescope optics, vibrations of the telescope, and so forth. Assuming that the optics are free of aberrations over the field of the CCD, this characteristic distribution of light, called the "Point Spread Function" (PSF) is the same for all point sources in the image. Usually, the PSF can be described by a symmetric Gaussian (bell-shaped) distribution (figure 1) [1]:

$$I_{(x,y)} = H \times e^{-\frac{(x-x_0)^2+(y-y_0)^2}{2\sigma^2}} + B \qquad (1)$$

$I_{(x,y)}$ is the intensity at the coordinates (x,y), which can be measured form the image. By fitting the PSF to the pixel values that make up the image of the object (figure 1), the quantities x_0, y_0, I, σ and B can be found, which characterize the point source as follows:

- Position
  The position of the object in the CCD frame can be expressed in rectangular coordinates (x_0, y_0), usually along the rows and columns of the CCD. Fitting a PSF to the image will allow to calculate the position of the object to a fraction of the pixel size.

- Intensity
  The height of the PSF (H) is proportional to the magnitude of the object. The total flux of the objects corresponds to the integrated volume of the PSF, less the background signal (see below).

- Width
  In equation 1, the width of the Gaussian PSF is characterized by the quantity σ. In astronomy, the width of the PSF is frequently specified by the so-called "Full Width Half Maximum" (FWHM). As the name implies, this is the width of the curve at half its height. The FWHM corresponds to approximately $2.355 \times \sigma$. Although a number of factors control the FWHM (like focusing, telescope optics, and vibrations), it is usually dominated by the seeing. The FWHM is the same for all point-sources in the image (if optical aberrations can be neglected). Most notably, it is independent of the brightness of the object. Bright stars appear larger on the image only because the faint outer extensions of the PSF are visible. For faint stars, these parts drown in the noise and are therefore not visible.

- Background
  During the exposure, the CCD not only collects signal from the object, but also light from the sky background and the thermal signal generated within the detector. These signals result in a pedestal (B) on which the PSF is based. Ideally, the background signal is the same over the whole field for calibrated images. In practice, however, it will vary somewhat over the field.

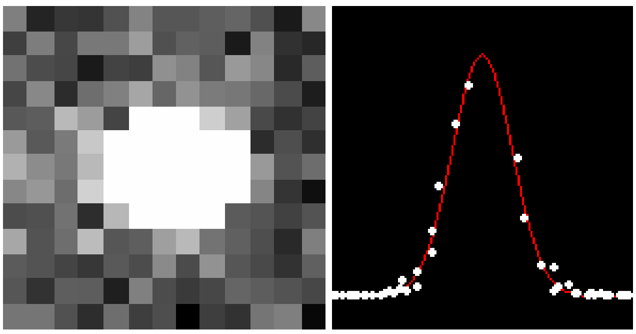

Figure 1: Image of a star on a CCD (left), and the Gaussian PSF fitted to the image data (right).

Signal and Noise

As briefly mentioned above, the CCD not only collects light from celestial objects, but also some unwanted signals. The thermal signal, for example, can be subtracted from the image by applying a dark frame calibration, but the noise of the thermal signal remains even in the calibrated image. In addition to the thermal noise, the readout noise is generated in the detector. External sources of noise are the photon noise in the signal from the sky background, as well as the photon noise in the signal of the object under observation. The Poisson noise in a signal (that is: the standard deviation σ of the individual measurements from the true signal) can be estimated as the square root of the signal, i.e.

$$\sigma = \sqrt{S} \qquad (2)$$

where S is the signal (for example, the thermal signal), and σ is the noise level in that signal (in that example, the thermal noise). The total noise from the four independent noise sources mentioned above add in quadrature to give the total noise:

$$\sigma = \sqrt{\sigma_B^2 + \sigma_S^2 + \sigma_T^2 + \sigma_R^2} \qquad (3)$$

where σ is the total noise, σ_B is the background noise, σ_s is the object noise, σ_T is the thermal signal, and σ_R is the readout noise. The Signal Noise Ratio (SNR) can be calculated from:

$$SNR = \frac{S}{\sigma} \qquad (4)$$

Where S is the signal from the object, and σ the total noise. By combining equations 2 to 4, it is possible to calculate the SNR in one pixel:

$$SNR = \frac{S}{\sqrt{S + B + T + \sigma_R^2}} \qquad (5)$$

Here, S is the signal from the object collected in the pixel, B the signal from the sky background and T the thermal signal collected by the pixel, respectively, and σ_R is the readout noise for one pixel. If equation 5 is applied to the brightest pixel in the image of the object, the result is the Peak SNR for that object. The Peak SNR is important, as software (or humans) can detect faint objects only if at least the brightest pixel has a SNR over some threshold that is set to avoid false detections in the image noise. Usually, a Peak SNR of ~3 is considered to be a marginal detection. In other words, this would correspond to the limiting magnitude of the image.

For unfiltered or broadband images, the dominant source of noise is usually the sky background, even under very dark skies. With modern, cooled CCDs, the instrumental noise is generally less important, and object noise is only significant for very bright objects.

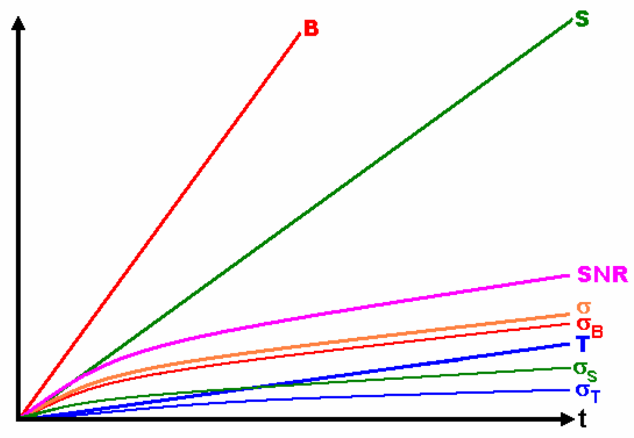

Figure 2: Growth of Signal, Noise, and Signal to Noise Ratio with increasing exposure time.

Figure 2 shows the growth of Signal, Noise, and Signal to Noise Ratio (SNR) with increasing exposure time t. Note that, in this example, the background signal B is stronger than the signal S from the object under observation. The background noise is σ_B, the object noise is σ_S. The readout noise is independent of the exposure time and it is therefore not drawn. (For sky-limited exposures, it can practically be neglected.) The signal S grows linear with increasing exposure time, as do the background signal B and the thermal signal T. Fortunately, the background noise ($\sigma_B = \sqrt{B}$) and the thermal noise ($\sigma_T = \sqrt{T}$) grow slower. Doubling the exposure time will increase all signals (S,B,T) by a factor of 2, but the noise levels (σ_s, σ_B, σ_T, σ) by a factor of only $\sqrt{2}$, so the SNR increases by $2 \div \sqrt{2} = \sqrt{2}$. With increasing exposure, the faint object will eventually emerge from the noise, even though the background signal is always stronger than the signal from the object in this example.

Estimating the Signal to Noise Ratio

With a few, mostly very simple calculations, it is possible to estimate the Signal to Noise Ratio that can be expected for a stellar object of known magnitude with a certain equipment. In this chapter, one example is described in some detail. Further examples in the following chapters will be used to compare various telescope setups, and the gain (or loss) in the SNR.

The telescope used in this example is a 0.6m f/3.3 reflector, with a central obstruction of 0.2m. As a detector, a CCD with 24µm square pixels (corresponding to 2.5” at the focal length of 1.98m), a dark current of one electron per second per pixel, a readout noise of ten electrons per pixel, and a mean quantum efficiency of 70% over the visible and near infrared portion of the spectrum (400nm to 800nm) is used [2]. We assume a stellar object of 20^{mag} as the target of the observation, the brightness of the sky background to be 18^{mag} per square arc second, and the FWHM of the stellar image to be 4”. In that spectral range, we receive about 4×10^{10} photons per second per square meter from a star of 0^{mag} [3]. A difference of 1^{mag} corresponds to a factor of 2.5 in the brightness, so there will be only $4\times10^{10} \div 2.5^{20}$, or about 440 photons per second per square meter from our target. The light collecting area of the 0.6m telescope is 0.25m², so it will accumulate 11’000 photons in a 100 second exposure. With a quantum efficiency of 0.7, this will generate about 7'700 electrons in the CCD.

Assuming that the PSF of the object can be described with equation 1, and that the peak brightness is located exactly at the centre of one pixel, this pixel collects about 29% of the total light, or about 3'190 photons, which will generate 2'233 electrons in that pixel. The Poisson noise of this signal is $\sqrt{2'233} \sim 47$.

In analogy to the stellar flux, we can estimate the flux from the sky background (18^{mag} per square arc second) to be $4{\times}10^{10} \div 2.5^{18}$, or about 2'748 photons per second per square meter. The telescope therefore collects about 68'700 photons from each square arc second during the exposure. Each pixel covers 6.25 square arc seconds, and therefore, about 429'375 photons from the sky background will be collected during the exposure in each pixel. This will generate about 300'563 electrons, with a Poisson noise of ~ 548 electrons.

During the exposure, the dark current will generate 100 electrons in each pixel, and the dark noise is therefore $\sqrt{100} = 10$. The readout adds further 10 noise electrons.

Using equation 3, the total noise in the brightest pixel can be calculated by adding the object noise in the brightest pixel, the sky noise, the dark noise and the readout noise in quadrature, i.e. $\sqrt{(47^2 + 548^2 + 10^2 + 10^2)} \sim 550$. The Peak Signal Noise Ratio is now found to be $2'233 \div 550 \sim 4.1$. Obviously, the 20^{mag} object is only marginally detected in this example.

Although this is a simplified calculation (e.g., no attempt to correct for atmospheric extinction was made, and no attention was given to the saturation of pixels, etc.), it is still a reasonable estimate. Some further telescope setups will be compared in the next chapters, and the results are compared. All calculations are summarized in table 1 in the Appendix.

Exposure Time

In the previous chapter, a star of 20^{mag} is only marginally detected with a 0.6m f/3.3 telescope in a 100 second exposure. In the next example, the exposure time is extended to 600 seconds to increase the Signal Noise Ratio of the object. The calculation, which is summarized as example 2 in table 1 in the Appendix, shows that the SNR of the brightest pixels increases from 4.1 to 10.0. It has been noted previously that increasing the exposure time by a factor of n will raise the SNR by a factor of $\sqrt{n}$. In this example, the exposure time has been increased by a factor of 6, and the SNR was raised by a factor of $\sqrt{6} \sim 2.45$.

The limiting magnitude of an image can be defined by the brightness of the stars reaching some minimal SNR, for example, 3.0. A factor of 2.5 in brightness corresponds to one magnitude, which closely matches the increase in SNR due to the longer exposure. To increase the limiting magnitude by one full magnitude, the exposure time would have to be extended by a factor of 6.25, i.e., to 625 seconds. Pushing the limiting magnitude down by one more magnitude, another increase by a factor of 6.25 would be necessary: the exposure time would increase to about 3900 seconds, or 65 minutes (figure 3).

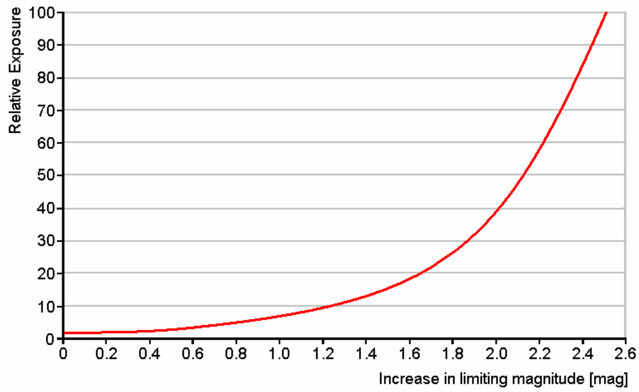

Figure 3: Relative exposure time required for increasing the limiting magnitude.

Telescope Aperture

In the next example, we will expand the telescope aperture from 0.6m (as used in the previous examples) to 1.5m, with a central obstruction of 0.5m in diameter and a focal length of 7m. For the environment (sky background, seeing) and the detector, the same values as in the previous examples are used, and a exposure time of 100 second (as in example 1) assumed. The result of the calculation, which is summarized as example 3 in table 1 in the Appendix, is somewhat surprising: Although the 1.5m telescope has 6.25 times more light collecting area than the 0.6m instrument, the Peak SNR is now only 3.4. Compared to the Peak SNR of 4.1 that was found for the 100 second integration with the 0.6m telescope, this is a loss of $\sim 0.2^{mag}$ in limiting magnitude.

How can this be? Due to the long focal length of the telescope, each pixel now covers only 0.71" × 0.71". Compared to the 0.6m telescope from the previous examples (pixel size 2.5" × 2.5"), this is only 8% of the area. By combining the increased light collecting power, and the smaller pixel scale, we find that each pixel receives only about $6.25 \times 0.08 \approx 0.5$ times the light collected in one pixel of the CCD by the smaller telescope. As both the light from the object and from the sky background (the dominant source of noise in these examples) drop by the factor of 0.5, the SNR should decrease approximately by a factor of $0.5 \div \sqrt{0.5} \sim 0.7$. The true factor found by comparing the SNR calculated in examples 1 and 3 is only about 0.8, because the PSF is a non-linear function (equation 1), concentrating more light in the centre of the pixel than in the outer regions that were lost due to the smaller angular size of the pixels in that example.

Does this mean that it makes no sense to use larger telescopes? Of course not! Apparently, the problem is related to the pixel scale, so pixel binning might be of some help: By using 2×2 binning (example 4), a Peak SNR of 6.4 is obtained, which corresponds to an increase in limiting magnitude of about 0.5^{mag} as compared to the 0.6m telescope in example 1, or of 0.7^{mag} as compared to the 1.5m telescope with the CCD used without binning (example 3).

Scaling the FWHM from 4" to 2" (by improving the telescope optics, the focusing, the mechanics or the seeing, if possible in some way) would be even better than binning: The peak SNR would grow to 12.7, and the gain in limiting magnitude is about 1.2^{mag}, as compared to example 1, or 1.4^{mag} as compared to example 3.

Pixel Size and Sampling

Apparently, the relative size of the pixel to the FWHM of the stellar images is an important factor in obtaining the highest possible SNR. By performing calculations similar to the SNR estimates in the previous chapters, it can be shown that the highest Peak SNR is obtained when the pixels are about $1.2 \times$ FWHM in size (figure 4).

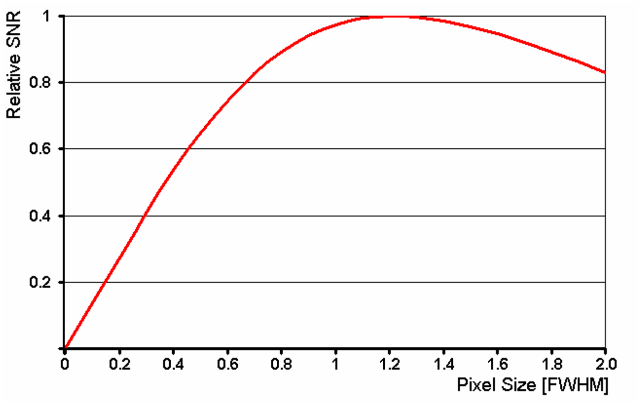

Figure 4: Variation of peak SNR for various pixel scales. The pixel size is measured in units of FWHM.

With such large pixels, most of the photons are collected by the single pixel on which the PSF of the stellar image is centred, whilst only the fainter, noisy "wings" of the PSF fall on the neighbouring pixels, resulting in a high SNR. However, with almost all the light concentrated in a single pixel, it would be very difficult to distinguish real objects from image artefacts (like hot pixels or cosmic ray strikes), and it is impossible to calculate the precise position of the object to sub-pixel accuracy.
To retain the information of the objects on the CCD image, the scale must be chosen so that the FWHM of stellar sources spans at least 1.5 to 2 pixels [4]. This scale is called "critical sampling", as it preserves just enough information that the original PSF can be restored by some software analysing the image. With even larger pixels (i.e., less than 1.5 pixels per FWHM), the PSF can not be restored with sufficient precision, and astrometric or photometric data reduction is inaccurate, or not possible at all. This situation is called "undersampling". In the other extreme ("oversampling") the light of the object is spread over many pixels: Although the PSF of stellar objects can be restored with high precision in this case, the SNR is decreased (figure 5).
Critically sampled images will give the highest SNR and deepest limiting magnitude possible with a given equipment in a certain exposure time, without loosing important information contained in the image. For applications that demand the highest possible astrometric or photometric precision, one might consider some oversampling. The same is true for "pretty pictures", as stars on critically sampled images look rather blocky.

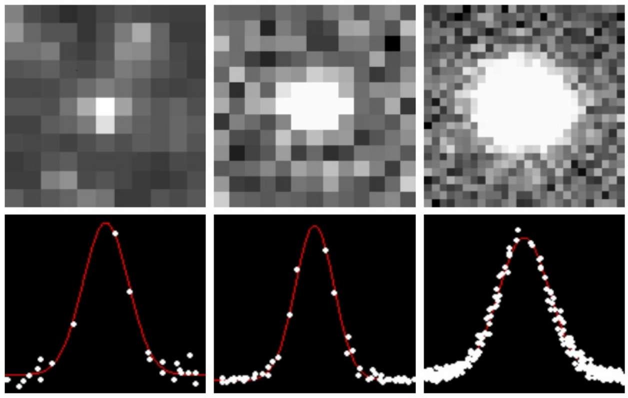

Figure 5: Undersampled (left), critically sampled (center) and oversampled (right) stellar images (top row), and the PSF fitted to the image data (bottom row).

Error Estimates

Fitting a PSF profile to a faint, noisy detection is naturally less precise than for bright stellar images with a high SNR (figure 6). Position and brightness calculated for faint detections are therefore expected to be less precise than for bright objects.

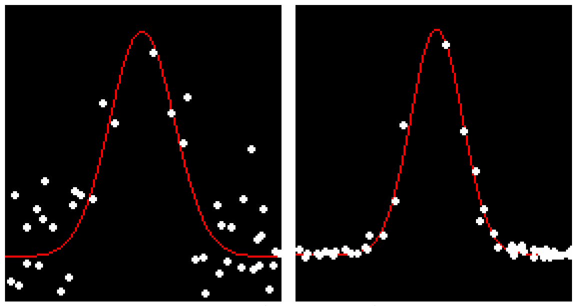

Figure 6: Gaussian PSF fitted to a faint (Peak SNR ~4) and a bright (Peak SNR ~100) stellar image.

The fractional uncertainty of the total flux is simply the reciprocal value of the Signal to Noise Ratio, $1 \div$ SNR (sometimes also called the Noise to Signal Ration). By converting this uncertainty to magnitudes, we get:

$$\sigma_{PHOT} = \frac{Log(1 + \frac{1}{SNR})}{Log(2.5)} \tag{6}$$

Here, σ_{PHOT} is the one-sigma random error estimated for the magnitude measured, and SNR is the total SNR of all pixels involved (e.g., within a synthetic aperture centred on the object). By modifying equation 5, we can find this value from:

$$SNR = \frac{S}{\sqrt{S + n \times \left(B + T + \sigma_R^2\right)}} \tag{7}$$

In this formula, S is the total integrated signal from the object in the measurement (i.e., within the aperture), and n is the number of pixels within the aperture. The other quantities are identical to equation 3. It should be noted that, as both S and n will change with the diameter of the aperture, the total SNR varies with the diameter of the photometric aperture, so photometry can be optimised by choosing the appropriate aperture [5].

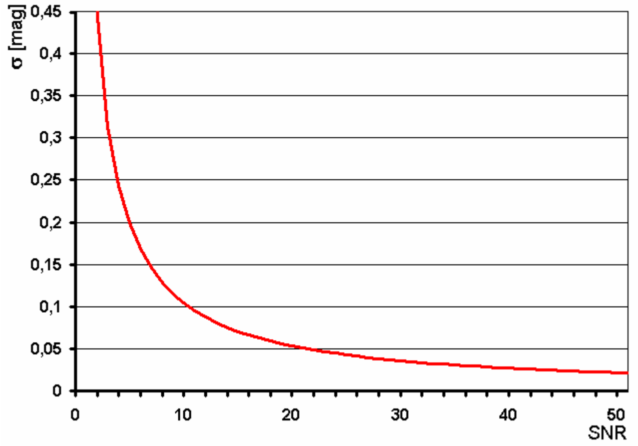

Figure 7: The photometric error (in stellar magnitudes) expected for point-sources up to a SNR of 50.

Figure 7 shows the expected uncertainty in the magnitude for point sources up to SNR 50, as calculated from equation 6. Equation 6 only estimates the random error in photometry due to image noise. It does not account for any systematic errors (like differences in spectral sensitivity of the CCD and the colour band used in the star catalogue) that might affect absolute photometric results.

Provided that the stellar images are properly sampled, the astrometric error can be estimated using this equation [6]:

$$\sigma_{AST} = \frac{\sigma_{PSF}}{SNR} \tag{8}$$

Here, σ_{AST} is the estimated one-sigma error of the position of the object, σ_{PSF} the Gaussian sigma of the PSF (as in equation 1), and SNR is the Peak Signal to Noise Ratio of the object. Note that σ_{AST} will be expressed in the same units as σ_{PSF} (usually arc seconds), and that σ_{PSF} can be calculated from FWHM÷2.355.

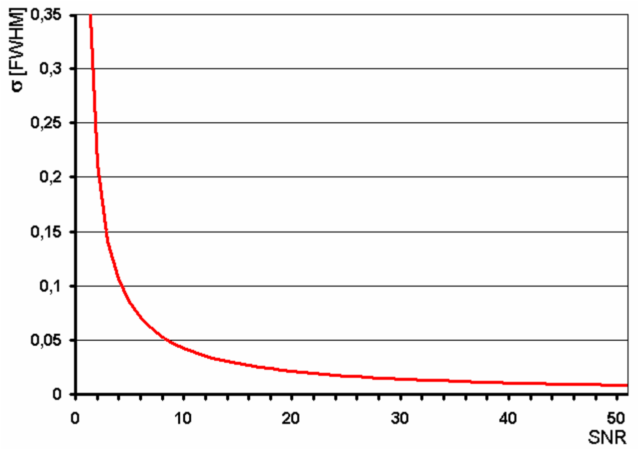

Figure 8: The astrometric error (in units of the FWHM) expected for point-sources up to a Peak SNR of 50.

Figure 8 shows the expected uncertainty in the position (in units of FWHM) for point sources up to SNR 50, as calculated from equation 8. Again, equation 8 only estimates the random error in the stellar centroid due to image noise. It does not account for any systematic errors (introduced by the astrometric reference star catalogue, for example) that might affect absolute astrometric results.
Returning to example 1, the astrometric one-sigma error expected for the point source with a Peak Signal to Noise Ratio of 4.1 and a FWHM of 4" can now be estimated to ~0.4", using equation 8. Adopting a photometric aperture with a diameter of 3 × FWHM (covering 18 pixels), a total Signal to Noise Ratio of about 3.3 is found by using equation 7. From equation 6, the photometric error is estimated to ~0.3^{mag}.
An astrometric error of ~1" is acceptable, particularly if it is a observation of a minor planet with a uncertain orbital solution or a large sky-plane uncertainty (for example, as in the case of late follow-up or recovery observations). Observations of the light curve of a minor planet usually require a precision of 0.05^{mag} or better, corresponding to a SNR of 20 or higher. Obviously, photometric observations are much more demanding than astrometry.

Summary and Conclusions

This paper first described the characteristics of a Gaussian Point Spread Function, and the sources of noise in the imaging system. A few examples, estimating the Signal to Noise Ratio obtained for faint point sources with various telescope setups, highlighted that environmental conditions, telescope equipment, and CCD detector must harmonise to operate at peak performance. Finally, the astrometric and photometric error expected when measuring faint point sources was estimated.
Useful astrometric results can be obtained even for very faint targets at the limit of detection, particularly if the sky-plane uncertainty for the object under observation is large. For photometric studies, a higher SNR is desirable.

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Appendix

Signal to Noise Ratio Estimation					
	Example 1	**Example 2**	**Example 3**	**Example 4**	**Example 5**
Telescope					
Mirror Diameter	0.60 m	0.60 m	1.50 m	1.50 m	1.50 m
Obstruction	0.20 m	0.20 m	0.50 m	0.50 m	0.50 m
Light Collecting Area	0.25 m²	0.25 m²	1.57 m²	1.57 m²	1.57 m²
Local Length	1.98 m	1.98 m	7.00 m	7.00 m	7.00 m
Focal Ratio	3.30	3.30	4.67	4.67	4.67
Detector					
Pixel Size	24 µm	24 µm	24 µm	48 µm	24 µm
Pixel Scale	2.50 ”/Pixel	2.50 ”/Pixel	0.71 ”/Pixel	1.42 ”/Pixel	0.71 ”/Pixel
Dark Current	1 e^-/s/Pixel	1 e^-/s/Pixel	1 e^-/s/Pixel	4 e^-/s/Pixel	1 e^-/s/Pixel
Readout Noise	10 e^-	10 e^-	10 e^-	20 e^-	10 e^-
Quantum Efficiency	70 %	70 %	70 %	70 %	70 %
Integration Time	100 s	600 s	100 s	100 s	100 s
Object and Sky					
Object Magnitude	20 ^{mag}	20 ^{mag}	20 ^{mag}	20 ^{mag}	20 ^{mag}
Sky Background	18 ^{mag}/☐”	18 ^{mag}/☐”	18 ^{mag}/☐”	19 ^{mag}/☐”	19 ^{mag}/☐”
FWHM	4”	4”	4”	4”	2”
SNR Calculation					
Object Flux	11'000 γ	66’000 γ	69’080 γ	69’080 γ	69’080 γ
Object Signal	7’700 e^-	46’200 e^-	48’356 e^-	48’356 e^-	48’356 e^-
Share for central Pixel	0.29	0.29	0.027	0.104	0.104
Object Flux in centr. Pixel	3’190 γ	19’140 γ	1’865 γ	7’184 γ	7’184 γ
Object Signal in centr. Pixel	2’233 e^-	13’394 e^-	1’305 e^-	5’029 e^-	5’029 e^-
Object Noise in centr. Pixel	47 e^-	116 e^-	36 e^-	71 e^-	71 e^-
Background Flux	429’375 γ/pixel	2'576’250 γ/pixel	217’487 γ/pixel	869’948 γ/pixel	217’487 γ/pixel
Background Signal	300’648 e^-/pixel	1’803’375 e^-/pixel	152’241 e^-/pixel	608’964 e^-/pixel	152’241 e^-/pixel
Background Noise	548 e^-/pixel	1342 e^-/pixel	390 e^-/pixel	780 e^-/pixel	390 e^-/pixel
Dark Current	100 e^-/pixel	600 e^-/pixel	100 e^-/pixel	400 e^-/pixel	100 e^-/pixel
Dark Noise	10 e^-/pixel	25 e^-/pixel	10 e^-/pixel	20 e^-/pixel	10 e^-/pixel
Noise in centr. Pixel	550 e^-	1342 e^-	392 e^-	783 e^-	397 e^-
Peak SNR	4.1	10.0	3.4	6.4	12.7

Table 1: Summary of the SNR calculations mentioned in the text. Example 1 is described in some detail in the paper. Note that, for example 4, the pixel size listed in the table is not the physical size, but the site of the 2×2 binned pixel, and all other data refer to the binned pixel.